



INDIANA COMMON CORE STATE STANDARDS  
**ALGEBRA I**

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## Introduction

The world we live in continues to change. For students to succeed in school, at work, and in the community, they will need more skills and knowledge than ever before. To ensure all students have every opportunity to succeed, Indiana adopted the Common Core State Standards in the area of Mathematics, as well as English Language Arts and Literacy in History/Social Studies, Science, and Technical Subjects.

## Common Core State Standards Philosophy

The Common Core State Standards:

- are aligned with college and work expectations.
- include rigorous content *and* application of knowledge through high-order skills.
- build upon strengths and lessons of current state standards.
- are internationally benchmarked.
- are evidence and/or research-based.

The Common Core State Standards will

- help prepare all students with the knowledge and skills needed to succeed in college and careers.
- assist students who move between states.
- create the opportunity for America to compete for high-wage, high-skill jobs in a knowledge-based economy.
- allow for more focused pre-service education and professional development.
- create economies of scale around areas such as curriculum, instructional resources, and assessment.

## Role of the Teacher

While the standards set expectations for student learning for each grade level, they do not prescribe how the standards should be taught. Teachers should use their skills, experience, talents, and resources to design standards-based classroom lessons that meet the needs of individual students.

## Assessment Horizon

Indiana is a governing state in the Partnership for Assessment of Readiness for College and Careers (PARCC). Through PARCC a new generation of assessments will be developed and used throughout the nation, allowing us to compare and evaluate policies that affect student achievement across states and school districts.

The Common Core assessment includes a summative test, which will be administered online beginning with the 2014-15 school year. The assessment also includes “through-course assessments” that take place over the course of the year, allowing teachers to adjust their instruction based on students’ results. As a governing state, Indiana will pilot Common Core items in 2012-13 and 2013-14 and provide schools with data regarding student performance on the Common Core State Standards.

## Impact on Instruction

### Grades K, 1, and 2

In 2011-12, kindergarten teachers will be the first to teach *only* the Common Core. Those kindergarteners will be the first class of 3<sup>rd</sup> graders to participate in the Common Core assessment. First-grade and second-grade students will learn Indiana Academic Standards and identified elements of Common Core in 2011-12. First-grade students will learn *only* Common Core in 2012-13. Second-grade students will learn *only* Common Core in 2013-14.

### Grades 3-12

Students in grades 3-12 will receive an intentional introduction to the Common Core beginning with the 2011-12 school year to ensure they have the foundation for future learning with no instructional gaps. In 2014-15, all ELA and math teachers will *only* teach Common Core.

## Final Notes

Indiana Department of Education staff members are working diligently, both externally with other states, and internally across the IDOE, to ensure the development of a common scope and sequence for Common Core. This work is critical as we begin the transition to the Common Core State Standards.

Equally as important, efforts are underway to assist teachers in understanding what to teach next year (and in subsequent years). A tight alignment between the content which is “taught” and that which is “tested” on ISTEP+ and ECAs, together with the initiation of Common Core State Standards instruction, facilitates the important work of developing the foundation for future learning and assessment.

## Traditional Pathway: High School Algebra I

The fundamental purpose of this course is to formalize and extend the mathematics that students learned in the middle grades. Because it is built on the middle grades standards, this is a more ambitious version of Algebra I than has generally been offered. The critical areas, called units, deepen and extend understanding of linear and exponential relationships by contrasting them with each other and by applying linear models to data that exhibit a linear trend, and students engage in methods for analyzing, solving, and using quadratic functions. The Mathematical Practice Standards apply throughout each course and, together with the content standards, prescribe that students experience mathematics as a coherent, useful, and logical subject that makes use of their ability to make sense of problem situations.

**Critical Area 1:** By the end of eighth grade, students have learned to solve linear equations in one variable and have applied graphical and algebraic methods to analyze and solve systems of linear equations in two variables. Now, students analyze and explain the process of solving an equation. Students develop fluency writing, interpreting, and translating between various forms of linear equations and inequalities, and using them to solve problems. They master the solution of linear equations and apply related solution techniques and the laws of exponents to the creation and solution of simple exponential equations.

**Critical Area 2:** In earlier grades, students define, evaluate, and compare functions, and use them to model relationships between quantities. In this unit, students will learn function notation and develop the concepts of domain and range. They explore many examples of functions, including sequences; they interpret functions given graphically, numerically, symbolically, and verbally, translate between representations, and understand the limitations of various representations. Students build on and informally extend their understanding of integer exponents to consider exponential functions. They compare and contrast linear and exponential functions, distinguishing between additive and multiplicative change. Students explore systems of equations and inequalities, and they find and interpret their solutions. They interpret arithmetic sequences as linear functions and geometric sequences as exponential functions.

**Critical Area 3:** This unit builds upon prior students' prior experiences with data, providing students with more formal means of assessing how a model fits data. Students use regression techniques to describe approximately linear relationships between quantities. They use graphical representations and knowledge of the context to make judgments about the appropriateness of linear models. With linear models, they look at residuals to analyze the goodness of fit.

**Critical Area 4:** In this unit, students build on their knowledge from unit 2, where they extended the laws of exponents to rational exponents. Students apply this new understanding of number and strengthen their ability to see structure in and create quadratic and exponential expressions. They create and solve equations, inequalities, and systems of equations involving quadratic expressions.

**Critical Area 5:** In this unit, students consider quadratic functions, comparing the key characteristics of quadratic functions to those of linear and exponential functions. They select from among these functions to model phenomena. Students learn to anticipate the graph of a quadratic function by interpreting various forms of quadratic expressions. In particular, they identify the real solutions of a quadratic equation as the zeros of a related quadratic function. Students expand their experience with functions to include more specialized functions—absolute value, step, and those that are piecewise-defined.

Units	Includes Standard Clusters*	Mathematical Practice Standards
<b>Unit 1</b> Relationships Between Quantities and Reasoning with Equations	<ul style="list-style-type: none"> <li>Reason quantitatively and use units to solve problems.</li> <li>Interpret the structure of expressions.</li> <li>Create equations that describe numbers or relationships.</li> <li>Understand solving equations as a process of reasoning and explain the reasoning.</li> <li>Solve equations and inequalities in one variable.</li> </ul>	<b>Make sense of problems and persevere in solving them.</b>  <b>Reason abstractly and quantitatively.</b>  <b>Construct viable arguments and critique the reasoning of others.</b>  <b>Model with mathematics.</b>  <b>Use appropriate tools strategically.</b>  <b>Attend to precision.</b>  <b>Look for and make use of structure.</b>  <b>Look for and express regularity in repeated reasoning.</b>
<b>Unit 2</b> Linear and Exponential Relationships	<ul style="list-style-type: none"> <li>Extend the properties of exponents to rational exponents.</li> <li>Solve systems of equations.</li> <li>Represent and solve equations and inequalities graphically.</li> <li>Understand the concept of a function and use function notation.</li> <li>Interpret functions that arise in applications in terms of a context.</li> <li>Analyze functions using different representations.</li> <li>Build a function that models a relationship between two quantities.</li> <li>Build new functions from existing functions.</li> <li>Construct and compare linear, quadratic, and exponential models and solve problems.</li> <li>Interpret expressions for functions in terms of the situation they model.</li> </ul>	
<b>Unit 3</b> Descriptive Statistics	<ul style="list-style-type: none"> <li>Summarize, represent, and interpret data on a single count or measurement variable.</li> <li>Summarize, represent, and interpret data on two categorical and quantitative variables.</li> <li>Interpret linear models.</li> </ul>	
<b>Unit 4</b> Expressions and Equations	<ul style="list-style-type: none"> <li>Interpret the structure of expressions.</li> <li>Write expressions in equivalent forms to solve problems.</li> <li>Perform arithmetic operations on polynomials.</li> <li>Create equations that describe numbers or relationships.</li> <li>Solve equations and inequalities in one variable.</li> <li>Solve systems of equations.</li> </ul>	
<b>Unit 5</b> Quadratic Functions and Modeling	<ul style="list-style-type: none"> <li>Use properties of rational and irrational numbers.</li> <li>Interpret functions that arise in applications in terms of a context.</li> <li>Analyze functions using different representations.</li> <li>Build a function that models a relationship between two quantities.</li> <li>Build new functions from existing functions.</li> <li>Construct and compare linear, quadratic, and exponential models and solve problems.</li> </ul>	

\*In some cases clusters appear in more than one unit within a course or in more than one course. Instructional notes will indicate how these standards grow over time. In some cases only certain standards within a cluster are included in a unit.

## Unit 1: Relationships Between Quantities and Reasoning with Equations

By the end of eighth grade students have learned to solve linear equations in one variable and have applied graphical and algebraic methods to analyze and solve systems of linear equations in two variables. This unit builds on these earlier experiences by asking students to analyze and explain the process of solving an equation. Students develop fluency writing, interpreting, and translating between various forms of linear equations and inequalities, and using them to solve problems. They master the solution of linear equations and apply related solution techniques and the laws of exponents to the creation and solution of simple exponential equations. All of this work is grounded on understanding quantities and on relationships between them.

Unit 1: Relationships between Quantities and Reasoning with Equations	
Clusters with Instructional Notes	Common Core State Standards
<p>SKILLS TO MAINTAIN</p> <p><i>Reinforce understanding of the properties of integer exponents. The initial experience with exponential expressions, equations, and functions involves integer exponents and builds on this understanding.*</i></p>	
<ul style="list-style-type: none"> <li>Reason quantitatively and use units to solve problems.</li> </ul> <p><i>Working with quantities and the relationships between them provides grounding for work with expressions, equations, and functions.</i></p>	<p>N.Q.1 Use units as a way to understand problems and to guide the solution of multi-step problems; choose and interpret units consistently in formulas; choose and interpret the scale and the origin in graphs and data displays.</p> <p>N.Q.2 Define appropriate quantities for the purpose of descriptive modeling.</p> <p>N.Q.3 Choose a level of accuracy appropriate to limitations on measurement when reporting quantities.</p>
<ul style="list-style-type: none"> <li>Interpret the structure of expressions.</li> </ul> <p><i>Limit to linear expressions and to exponential expressions with integer exponents.</i></p>	<p>A.SSE.1 Interpret expressions that represent a quantity in terms of its context.*</p> <ol style="list-style-type: none"> <li>Interpret parts of an expression, such as terms, factors, and coefficients.</li> <li>Interpret complicated expressions by viewing one or more of their parts as a single entity. <i>For example, interpret <math>P(1+r)^n</math> as the product of <math>P</math> and a factor not depending on <math>P</math>.</i></li> </ol>
<ul style="list-style-type: none"> <li>Create equations that describe numbers or relationships.</li> </ul> <p><i>Limit A.CED.1 and A.CED.2 to linear and exponential equations, and, in the case of exponential equations, limit to situations requiring evaluation of exponential functions at integer inputs. Limit A.CED.3 to linear equations and inequalities. Limit A.CED.4 to formulas which are linear in the variable of interest.</i></p>	<p>A.CED.1 Create equations and inequalities in one variable and use them to solve problems. <i>Include equations arising from linear and quadratic functions, and simple rational and exponential functions.</i></p> <p>A.CED.2 Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales.</p> <p>A.CED.3 Represent constraints by equations or inequalities, and by systems of equations and/or inequalities, and interpret solutions as viable or non-viable options in a modeling context. <i>For example, represent inequalities describing nutritional and cost constraints on combinations of different foods.</i></p> <p>A.CED.4 Rearrange formulas to highlight a quantity of interest, using the same reasoning as in solving equations. <i>For example, rearrange Ohm's law <math>V = IR</math> to highlight resistance <math>R</math>.</i></p>

\*Instructional suggestions will be found in italics in this column throughout the document.

Unit 1: Relationships between Quantities and Reasoning with Equations	
Clusters with Instructional Notes	Common Core State Standards
<ul style="list-style-type: none"> <li>Understand solving equations as a process of reasoning and explain the reasoning.</li> </ul> <p><i>Students should focus on and master A.REI.1 for linear equations and be able to extend and apply their reasoning to other types of equations in future courses. Students will solve exponential equations with logarithms in Algebra II.</i></p>	A.REI.1 Explain each step in solving a simple equation as following from the equality of numbers asserted at the previous step, starting from the assumption that the original equation has a solution. Construct a viable argument to justify a solution method.
<ul style="list-style-type: none"> <li>Solve equations and inequalities in one variable.</li> </ul> <p><i>Extend earlier work with solving linear equations to solving linear inequalities in one variable and to solving literal equations that are linear in the variable being solved for. Include simple exponential equations that rely only on application of the laws of exponents, such as <math>5^x=125</math> or <math>2^x=1/16</math>.</i></p>	A.REI.3 Solve linear equations and inequalities in one variable, including equations with coefficients represented by letters.

## Unit 2: Linear and Exponential Relationships

In earlier grades, students define, evaluate, and compare functions, and use them to model relationships between quantities. In this unit, students will learn function notation and develop the concepts of domain and range. They move beyond viewing functions as processes that take inputs and yield outputs and start viewing functions as objects in their own right. They explore many examples of functions, including sequences; they interpret functions given graphically, numerically, symbolically, and verbally, translate between representations, and understand the limitations of various representations. They work with functions given by graphs and tables, keeping in mind that, depending upon the context, these representations are likely to be approximate and incomplete. Their work includes functions that can be described or approximated by formulas as well as those that cannot. When functions describe relationships between quantities arising from a context, students reason with the units in which those quantities are measured. Students explore systems of equations and inequalities, and they find and interpret their solutions. Students build on and informally extend their understanding of integer exponents to consider exponential functions. They compare and contrast linear and exponential functions, distinguishing between additive and multiplicative change. They interpret arithmetic sequences as linear functions and geometric sequences as exponential functions.

Unit 2: Linear and Exponential Relationships	
Clusters with Instructional Notes	Common Core State Standards
<ul style="list-style-type: none"> <li>Extend the properties of exponents to rational exponents.</li> </ul> <p><i>In implementing the standards in curriculum, these standards should occur before discussing exponential functions with continuous domains.</i></p>	<p>N.RN.1 Explain how the definition of the meaning of rational exponents follows from extending the properties of integer exponents to those values, allowing for a notation for radicals in terms of rational exponents. <i>For example, we define <math>5^{1/3}</math> to be the cube root of 5 because we want <math>(5^{1/3})^3 = 5^{(1/3) \cdot 3}</math> to hold, so <math>(5^{1/3})^3</math> must equal 5.</i></p> <p>N.RN.2 Rewrite expressions involving radicals and rational exponents using the properties of exponents.</p>
<ul style="list-style-type: none"> <li>Solve systems of equations.</li> </ul> <p><i>Build on student experiences graphing and solving systems of linear equations from middle school to focus on justification of the methods used. Include cases where the two equations describe the same line (yielding infinitely many solutions) and cases where two equations describe parallel lines (yielding no solution); connect to GPE.5 when it is taught in Geometry, which requires students to prove the slope criteria for parallel lines.</i></p>	<p>A.REI.5 Prove that, given a system of two equations in two variables, replacing one equation by the sum of that equation and a multiple of the other produces a system with the same solutions.</p> <p>A.REI.6 Solve systems of linear equations exactly and approximately (e.g., with graphs), focusing on pairs of linear equations in two variables.</p>
<ul style="list-style-type: none"> <li>Represent and solve equations and inequalities graphically.</li> </ul> <p><i>For A.REI.10, focus on linear and exponential equations and be able to adapt and apply that learning to other types of equations in future courses. For A.REI.11, focus on cases where <math>f(x)</math> and <math>g(x)</math> are linear or exponential.</i></p>	<p>A.REI.10 Understand that the graph of an equation in two variables is the set of all its solutions plotted in the coordinate plane, often forming a curve (which could be a line).</p> <p>A.REI.11 Explain why the x-coordinates of the points where the graphs of the equations <math>y = f(x)</math> and <math>y = g(x)</math> intersect are the solutions of the equation <math>f(x) = g(x)</math>; find the solutions approximately, e.g., using technology to graph the functions, make tables of values, or find successive approximations. Include cases where <math>f(x)</math> and/or <math>g(x)</math> are linear, polynomial, rational, absolute value, exponential, and logarithmic functions.*</p> <p>A.REI.12 Graph the solutions to a linear inequality in two variables as a half-plane (excluding the boundary in the case of a strict inequality), and graph the solution set to a system of linear inequalities in two variables as the intersection of the corresponding half-planes.</p>



Unit 2: Linear and Exponential Relationships	
Clusters with Instructional Notes	Common Core State Standards
<ul style="list-style-type: none"> <li>Understand the concept of a function and use function notation.</li> </ul> <p><i>Students should experience a variety of types of situations modeled by functions. Detailed analysis of any particular class of functions at this stage is not advised. Students should apply these concepts throughout their future mathematics courses.</i></p> <p><i>Draw examples from linear and exponential functions. In F.IF.3, draw connection to F.BF.2, which requires students to write arithmetic and geometric sequences. Emphasize arithmetic and geometric sequences as examples of linear and exponential functions.</i></p>	<p>F.IF.1 Understand that a function from one set (called the domain) to another set (called the range) assigns to each element of the domain exactly one element of the range. If <math>f</math> is a function and <math>x</math> is an element of its domain, then <math>f(x)</math> denotes the output of <math>f</math> corresponding to the input <math>x</math>. The graph of <math>f</math> is the graph of the equation <math>y = f(x)</math>.</p> <p>F.IF.2 Use function notation, evaluate functions for inputs in their domains, and interpret statements that use function notation in terms of a context.</p> <p>F.IF.3 Recognize that sequences are functions, sometimes defined recursively, whose domain is a subset of the integers. <i>For example, the Fibonacci sequence is defined recursively by <math>f(0) = f(1) = 1</math>, <math>f(n+1) = f(n) + f(n-1)</math> for <math>n \geq 1</math>.</i></p>
<ul style="list-style-type: none"> <li>Interpret functions that arise in applications in terms of a context.</li> </ul> <p><i>For F.IF.4 and 5, focus on linear and exponential functions. For F.IF.6, focus on linear functions and exponential functions whose domain is a subset of the integers. Unit 5 in this course and the Algebra II course address other types of functions.</i></p>	<p>F.IF.4 For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. <i>Key features include: intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; end behavior; and periodicity.</i>★</p> <p>F.IF.5 Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes. <i>For example, if the function <math>h(n)</math> gives the number of person-hours it takes to assemble <math>n</math> engines in a factory, then the positive integers would be an appropriate domain for the function.</i>★</p> <p>F.IF.6 Calculate and interpret the average rate of change of a function (presented symbolically or as a table) over a specified interval. Estimate the rate of change from a graph.★</p>
<ul style="list-style-type: none"> <li>Analyze functions using different representations.</li> </ul> <p><i>For F.IF.7a, 7e, and 9 focus on linear and exponentials functions. Include comparisons of two functions presented algebraically. For example, compare the growth of two linear functions, or two exponential functions such as <math>y=3^n</math> and <math>y=100^2</math></i></p>	<p>F.IF.7 Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases.★</p> <ul style="list-style-type: none"> <li>a. Graph linear and quadratic functions and show intercepts, maxima, and minima.</li> <li>e. Graph exponential and logarithmic functions, showing intercepts and end behavior, and trigonometric functions, showing period, midline, and amplitude.</li> </ul> <p>F.IF.9 Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions). <i>For example, given a graph of one quadratic function and an algebraic expression for another, say which has the larger maximum.</i></p>

Unit 2: Linear and Exponential Relationships	
Clusters with Instructional Notes	Common Core State Standards
<ul style="list-style-type: none"> <li>Build a function that models a relationship between two quantities.</li> </ul> <p><i>Limit to F.BF.1a, 1b, and 2 to linear and exponential functions. In F.BF.2, connect arithmetic sequences to linear functions and geometric sequences to exponential functions.</i></p>	<p>F.BF.1 Write a function that describes a relationship between two quantities.*</p> <ol style="list-style-type: none"> <li>Determine an explicit expression, a recursive process, or steps for calculation from a context.</li> <li>Combine standard function types using arithmetic operations. <i>For example, build a function that models the temperature of a cooling body by adding a constant function to a decaying exponential, and relate these functions to the model.</i></li> </ol> <p>F.BF.2 Write arithmetic and geometric sequences both recursively and with an explicit formula, use them to model situations, and translate between the two forms.*</p>
<ul style="list-style-type: none"> <li>Build new functions from existing functions.</li> </ul> <p><i>Focus on vertical translations of graphs of linear and exponential functions. Relate the vertical translation of a linear function to its y-intercept.</i></p> <p><i>While applying other transformations to a linear graph is appropriate at this level, it may be difficult for students to identify or distinguish between the effects of the other transformations included in this standard.</i></p>	<p>F.BF.3 Identify the effect on the graph of replacing <math>f(x)</math> by <math>f(x) + k</math>, <math>k f(x)</math>, <math>f(kx)</math>, and <math>f(x + k)</math> for specific values of <math>k</math> (both positive and negative); find the value of <math>k</math> given the graphs. Experiment with cases and illustrate an explanation of the effects on the graph using technology. <i>Include recognizing even and odd functions from their graphs and algebraic expressions for them.</i></p>
<ul style="list-style-type: none"> <li>Construct and compare linear, quadratic, and exponential models and solve problems.</li> </ul> <p><i>For F.LE.3, limit to comparisons between linear and exponential models. In constructing linear functions in F.LE.2, draw on and consolidate previous work in Grade 8 on finding equations for lines and linear functions (8.EE.6, 8.F.4).</i></p>	<p>F.LE.1 Distinguish between situations that can be modeled with linear functions and with exponential functions.</p> <ol style="list-style-type: none"> <li>Prove that linear functions grow by equal differences over equal intervals; and that exponential functions grow by equal factors over equal intervals.</li> <li>Recognize situations in which one quantity changes at a constant rate per unit interval relative to another.</li> <li>Recognize situations in which a quantity grows or decays by a constant percent rate per unit interval relative to another.</li> </ol> <p>F.LE.2 Construct linear and exponential functions, including arithmetic and geometric sequences, given a graph, a description of a relationship, or two input-output pairs (include reading these from a table).</p> <p>F.LE.3 Observe using graphs and tables that a quantity increasing exponentially eventually exceeds a quantity increasing linearly, quadratically, or (more generally) as a polynomial function.</p>
<ul style="list-style-type: none"> <li>Interpret expressions for functions in terms of the situation they model.</li> </ul> <p><i>Limit exponential functions to those of the form <math>f(x) = b^x + k</math>.</i></p>	<p>F.LE.5 Interpret the parameters in a linear or exponential function in terms of a context.</p>

## Unit 3: Descriptive Statistics

Experience with descriptive statistics began as early as Grade 6. Students were expected to display numerical data and summarize it using measures of center and variability. By the end of middle school they were creating scatter-plots and recognizing linear trends in data. This unit builds upon that prior experience, providing students with more formal means of assessing how a model fits data. Students use regression techniques to describe approximately linear relationships between quantities. They use graphical representations and knowledge of the context to make judgments about the appropriateness of linear models. With linear models, they look at residuals to analyze the goodness of fit.

Unit 3: Descriptive Statistics	
Clusters with Instructional Notes	Common Core State Standards
<ul style="list-style-type: none"> <li>Summarize, represent, and interpret data on a single count or measurement variable.</li> </ul> <p><i>In grades 6 – 8, students describe center and spread in a data distribution. Here they choose a summary statistic appropriate to the characteristics of the data distribution, such as the shape of the distribution or the existence of extreme data points.</i></p>	<p>S.ID.1 Represent data with plots on the real number line (dot plots, histograms, and box plots).</p> <p>S.ID.2 Use statistics appropriate to the shape of the data distribution to compare center (median, mean) and spread (interquartile range, standard deviation) of two or more different data sets.</p> <p>S.ID.3 Interpret differences in shape, center, and spread in the context of the data sets, accounting for possible effects of extreme data points (outliers).</p>
<ul style="list-style-type: none"> <li>Summarize, represent, and interpret data on two categorical and quantitative variables.</li> </ul> <p><i>Students take a more sophisticated look at using a linear function to model the relationship between two numerical variables. In addition to fitting a line to data, students assess how well the model fits by analyzing residuals.</i></p> <p><i>S.ID.6b should be focused on linear models, but may be used to preview quadratic functions in Unit 5 of this course.</i></p>	<p>S.ID.5 Summarize categorical data for two categories in two-way frequency tables. Interpret relative frequencies in the context of the data (including joint, marginal, and conditional relative frequencies). Recognize possible associations and trends in the data.</p> <p>S.ID.6 Represent data on two quantitative variables on a scatter plot, and describe how the variables are related.</p> <ol style="list-style-type: none"> <li>Fit a function to the data; use functions fitted to data to solve problems in the context of the data. <i>Use given functions or choose a function suggested by the context. Emphasize linear and exponential models.</i></li> <li>Informally assess the fit of a function by plotting and analyzing residuals.</li> <li>Fit a linear function for a scatter plot that suggests a linear association.</li> </ol>
<ul style="list-style-type: none"> <li>Interpret linear models.</li> </ul> <p><i>Build on students' work with linear relationships in eighth grade and introduce the correlation coefficient. The focus here is on the computation and interpretation of the correlation coefficient as a measure of how well the data fit the relationship. The important distinction between a statistical relationship and a cause-and-effect relationship arises in S.ID.9.</i></p>	<p>S.ID.7 Interpret the slope (rate of change) and the intercept (constant term) of a linear model in the context of the data.</p> <p>S.ID.8 Compute (using technology) and interpret the correlation coefficient of a linear fit.</p> <p>S.ID.9 Distinguish between correlation and causation.</p>

## Unit 4: Expressions and Equations

In this unit, students build on their knowledge from unit 2, where they extended the laws of exponents to rational exponents. Students apply this new understanding of number and strengthen their ability to see structure in and create quadratic and exponential expressions. They create and solve equations, inequalities, and systems of equations involving quadratic expressions.

Unit 4: Expressions and Equations	
Clusters with Instructional Notes	Common Core State Standards
<ul style="list-style-type: none"> <li>Interpret the structure of expressions.</li> </ul> <p><i>Focus on quadratic and exponential expressions. For A.SSE.1b, exponents are extended from the integer exponents found in Unit 1 to rational exponents focusing on those that represent square or cube roots.</i></p>	<p>A.SSE.1 Interpret expressions that represent a quantity in terms of its context.*</p> <ol style="list-style-type: none"> <li>Interpret parts of an expression, such as terms, factors, and coefficients.</li> <li>Interpret complicated expressions by viewing one or more of their parts as a single entity. <i>For example, interpret <math>P(1+r)^n</math> as the product of <math>P</math> and a factor not depending on <math>P</math>.</i></li> </ol> <p>A.SSE.2 Use the structure of an expression to identify ways to rewrite it. <i>For example, see <math>x^4 - y^4</math> as <math>(x^2)^2 - (y^2)^2</math>, thus recognizing it as a difference of squares that can be factored as <math>(x^2 - y^2)(x^2 + y^2)</math>.</i></p>
<ul style="list-style-type: none"> <li>Write expressions in equivalent forms to solve problems.</li> </ul> <p><i>It is important to balance conceptual understanding and procedural fluency in work with equivalent expressions. For example, development of skill in factoring and completing the square goes hand-in-hand with understanding what different forms of a quadratic expression reveal.</i></p>	<p>A.SSE.3 Choose and produce an equivalent form of an expression to reveal and explain properties of the quantity represented by the expression.*</p> <ol style="list-style-type: none"> <li>Factor a quadratic expression to reveal the zeros of the function it defines.</li> <li>Complete the square in a quadratic expression to reveal the maximum or minimum value of the function it defines.</li> <li>Use the properties of exponents to transform expressions for exponential functions. <i>For example the expression <math>1.15^t</math> can be rewritten as <math>(1.15^{1/12})^{12t} \approx 1.012^{12t}</math> to reveal the approximate equivalent monthly interest rate if the annual rate is 15%.</i></li> </ol>
<ul style="list-style-type: none"> <li>Perform arithmetic operations on polynomials.</li> </ul> <p><i>Focus on polynomial expressions that simplify to forms that are linear or quadratic in a positive integer power of <math>x</math>.</i></p>	<p>A.APR.1 Understand that polynomials form a system analogous to the integers, namely, they are closed under the operations of addition, subtraction, and multiplication; add, subtract, and multiply polynomials.</p>
<ul style="list-style-type: none"> <li>Create equations that describe numbers or relationships.</li> </ul> <p><i>Extend work on linear and exponential equations in Unit 1 to quadratic equations. Extend A.CED.4 to formulas involving squared variables.</i></p>	<p>A.CED.1 Create equations and inequalities in one variable and use them to solve problems. <i>Include equations arising from linear and quadratic functions, and simple rational and exponential functions.</i></p> <p>A.CED.2 Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales.</p> <p>A.CED.4 Rearrange formulas to highlight a quantity of interest, using the same reasoning as in solving equations. <i>For example, rearrange Ohm's law <math>V = IR</math> to highlight resistance <math>R</math>.</i></p>
<ul style="list-style-type: none"> <li>Solve equations and inequalities in one variable.</li> </ul> <p><i>Students should learn of the existence of the complex number system, but will not solve quadratics with complex solutions until Algebra II.</i></p>	<p>A.REI.4 Solve quadratic equations in one variable.</p> <ol style="list-style-type: none"> <li>Use the method of completing the square to transform any quadratic equation in <math>x</math> into an equation of the form <math>(x - p)^2 = q</math> that has the same solutions. Derive the quadratic formula from this form.</li> <li>Solve quadratic equations by inspection (e.g., for <math>x^2 = 49</math>), taking square roots, completing the square, the quadratic formula and factoring, as appropriate to the initial form of the equation. Recognize when the quadratic formula gives complex solutions and write them as <math>a \pm bi</math> for real numbers <math>a</math> and <math>b</math>.</li> </ol>

Unit 4: Expressions and Equations	
Clusters with Instructional Notes	Common Core State Standards
<ul style="list-style-type: none"> <li>Solve systems of equations.</li> </ul> <p><i>Include systems consisting of one linear and one quadratic equation. Include systems that lead to work with fractions. For example, finding the intersections between <math>x^2 + y^2 = 1</math> and <math>y = (x+1)/2</math> leads to the point <math>(3/5, 4/5)</math> on the unit circle, corresponding to the Pythagorean triple <math>3^2 + 4^2 = 5^2</math>.</i></p>	<p>A.REI.7 Solve a simple system consisting of a linear equation and a quadratic equation in two variables algebraically and graphically. <i>For example, find the points of intersection between the line <math>y = -3x</math> and the circle <math>x^2 + y^2 = 3</math>.</i></p>

## Unit 5: Quadratic Functions and Modeling

In preparation for work with quadratic relationships students explore distinctions between rational and irrational numbers. They consider quadratic functions, comparing the key characteristics of quadratic functions to those of linear and exponential functions. They select from among these functions to model phenomena. Students learn to anticipate the graph of a quadratic function by interpreting various forms of quadratic expressions. In particular, they identify the real solutions of a quadratic equation as the zeros of a related quadratic function. Students learn that when quadratic equations do not have real solutions the number system must be extended so that solutions exist, analogous to the way in which extending the whole numbers to the negative numbers allows  $x+1=0$  to have a solution. Formal work with complex numbers comes in Algebra II. Students expand their experience with functions to include more specialized functions—absolute value, step, and those that are piecewise-defined.

Unit 5: Quadratic Functions and Modeling	
Clusters with Instructional Notes	Common Core State Standards
<ul style="list-style-type: none"> <li>Use properties of rational and irrational numbers.</li> </ul> <p><i>Connect N.RN.3 to physical situations, e.g., finding the perimeter of a square of area 2.</i></p>	<p>N.RN.3 Explain why the sum or product of two rational numbers is rational; that the sum of a rational number and an irrational number is irrational; and that the product of a nonzero rational number and an irrational number is irrational.</p>
<ul style="list-style-type: none"> <li>Interpret functions that arise in applications in terms of a context.</li> </ul> <p><i>Focus on quadratic functions; compare with linear and exponential functions studied in Unit 2.</i></p>	<p>F.IF.4 For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. <i>Key features include: intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; end behavior; and periodicity.</i>★</p> <p>F.IF.5 Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes. <i>For example, if the function <math>h(n)</math> gives the number of person-hours it takes to assemble <math>n</math> engines in a factory, then the positive integers would be an appropriate domain for the function.</i>★</p> <p>F.IF.6 Calculate and interpret the average rate of change of a function (presented symbolically or as a table) over a specified interval. Estimate the rate of change from a graph.★</p>
<ul style="list-style-type: none"> <li>Analyze functions using different representations.</li> </ul> <p><i>For F.IF.7b, compare and contrast absolute value, step and piecewise-defined functions with linear, quadratic, and exponential functions. Highlight issues of domain, range, and usefulness when examining piecewise-defined functions. Note that this unit, and in particular in F.IF.8b, extends the work begun in Unit 2 on exponential functions with integer exponents. For F.IF.9, focus on expanding the types of functions considered to include, linear, exponential, and quadratic.</i></p> <p><i>Extend work with quadratics to include the relationship between coefficients and roots, and that once roots are known, a quadratic equation can be factored.</i></p>	<p>F.IF.7 Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases.★</p> <ol style="list-style-type: none"> <li>Graph linear and quadratic functions and show intercepts, maxima, and minima.</li> <li>Graph square root, cube root, and piecewise-defined functions, including step functions and absolute value functions.</li> </ol> <p>F.IF.8 Write a function defined by an expression in different but equivalent forms to reveal and explain different properties of the function.</p> <ol style="list-style-type: none"> <li>Use the process of factoring and completing the square in a quadratic function to show zeros, extreme values, and symmetry of the graph, and interpret these in terms of a context.</li> <li>Use the properties of exponents to interpret expressions for exponential functions. <i>For example, identify percent rate of change in functions such as <math>y = (1.02)^t</math>, <math>y = (0.97)^t</math>, <math>y = (1.01)^{2t}</math>, <math>y = (1.2)^{t/10}</math>, and classify them as representing exponential growth or decay.</i></li> </ol> <p>F.IF.9 Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions). <i>For example, given a graph of one quadratic function and an algebraic expression for another, say which has the larger maximum.</i></p>

Unit 5: Quadratic Functions and Modeling	
Clusters with Instructional Notes	Common Core State Standards
<ul style="list-style-type: none"> <li>Build a function that models a relationship between two quantities.</li> </ul> <p><i>Focus on situations that exhibit a quadratic relationship.</i></p>	<p>F.BF.1 Write a function that describes a relationship between two quantities.*</p> <ol style="list-style-type: none"> <li>Determine an explicit expression, a recursive process, or steps for calculation from a context.</li> <li>Combine standard function types using arithmetic operations. <i>For example, build a function that models the temperature of a cooling body by adding a constant function to a decaying exponential, and relate these functions to the model.</i></li> </ol>
<ul style="list-style-type: none"> <li>Build new functions from existing functions.</li> </ul> <p><i>For F.BF.3, focus on quadratic functions, and consider including absolute value functions. For F.BF.4a, focus on linear functions but consider simple situations where the domain of the function must be restricted in order for the inverse to exist, such as <math>f(x) = x^2</math>, <math>x &gt; 0</math>.</i></p>	<p>F.BF.3 Identify the effect on the graph of replacing <math>f(x)</math> by <math>f(x) + k</math>, <math>k f(x)</math>, <math>f(kx)</math>, and <math>f(x + k)</math> for specific values of <math>k</math> (both positive and negative); find the value of <math>k</math> given the graphs. Experiment with cases and illustrate an explanation of the effects on the graph using technology. <i>Include recognizing even and odd functions from their graphs and algebraic expressions for them.</i></p> <p>F.BF.4 Find inverse functions.</p> <ol style="list-style-type: none"> <li>Solve an equation of the form <math>f(x) = c</math> for a simple function <math>f</math> that has an inverse and write an expression for the inverse. <i>For example, <math>f(x) = 2x^3</math> or <math>f(x) = (x+1)/(x-1)</math> for <math>x \neq 1</math>.</i></li> </ol>
<ul style="list-style-type: none"> <li>Construct and compare linear, quadratic, and exponential models and solve problems.</li> </ul> <p><i>Compare linear and exponential growth to quadratic growth.</i></p>	<p>F.LE.3 Observe using graphs and tables that a quantity increasing exponentially eventually exceeds a quantity increasing linearly, quadratically, or (more generally) as a polynomial function.</p>

# Mathematics | Standards for Mathematical Practice

The Standards for Mathematical Practice describe varieties of expertise that mathematics educators at all levels should seek to develop in their students. These practices rest on important “processes and proficiencies” with longstanding importance in mathematics education. The first of these are the NCTM process standards of problem solving, reasoning and proof, communication, representation, and connections. The second are the strands of mathematical proficiency specified in the National Research Council’s report *Adding It Up*: adaptive reasoning, strategic competence, conceptual understanding (comprehension of mathematical concepts, operations and relations), procedural fluency (skill in carrying out procedures flexibly, accurately, efficiently and appropriately), and productive disposition (habitual inclination to see mathematics as sensible, useful, and worthwhile, coupled with a belief in diligence and one’s own efficacy).

## 1 Make sense of problems and persevere in solving them.

Mathematically proficient students start by explaining to themselves the meaning of a problem and looking for entry points to its solution. They analyze givens, constraints, relationships, and goals. They make conjectures about the form and meaning of the solution and plan a solution pathway rather than simply jumping into a solution attempt. They consider analogous problems, and try special cases and simpler forms of the original problem in order to gain insight into its solution. They monitor and evaluate their progress and change course if necessary. Older students might, depending on the context of the problem, transform algebraic expressions or change the viewing window on their graphing calculator to get the information they need. Mathematically proficient students can explain correspondences between equations, verbal descriptions, tables, and graphs or draw diagrams of important features and relationships, graph data, and search for regularity or trends. Younger students might rely on using concrete objects or pictures to help conceptualize and solve a problem. Mathematically proficient students check their answers to problems using a different method, and they continually ask themselves, “Does this make sense?” They can understand the approaches of others to solving complex problems and identify correspondences between different approaches.

## 2 Reason abstractly and quantitatively.

Mathematically proficient students make sense of quantities and their relationships in problem situations. They bring two complementary abilities to bear on problems involving quantitative relationships: the ability to *decontextualize*—to abstract a given situation and represent it symbolically and manipulate the representing symbols as if they have a life of their own, without necessarily attending to their referents—and the ability to *contextualize*, to pause as needed during the manipulation process in order to probe into the referents for the symbols involved. Quantitative reasoning entails habits of creating a coherent representation of the problem at hand; considering the units involved; attending to the meaning of quantities, not just how to compute them; and knowing and flexibly using different properties of operations and objects.

## 3 Construct viable arguments and critique the reasoning of others.

Mathematically proficient students understand and use stated assumptions, definitions, and previously established results in constructing arguments. They make conjectures and build a logical progression of statements to explore the truth of their conjectures. They are able to analyze situations by breaking them into cases, and can recognize and use counterexamples. They justify their conclusions,



communicate them to others, and respond to the arguments of others. They reason inductively about data, making plausible arguments that take into account the context from which the data arose. Mathematically proficient students are also able to compare the effectiveness of two plausible arguments, distinguish correct logic or reasoning from that which is flawed, and—if there is a flaw in an argument—explain what it is. Elementary students can construct arguments using concrete referents such as objects, drawings, diagrams, and actions. Such arguments can make sense and be correct, even though they are not generalized or made formal until later grades. Later, students learn to determine domains to which an argument applies. Students at all grades can listen or read the arguments of others, decide whether they make sense, and ask useful questions to clarify or improve the arguments.

#### **4 Model with mathematics.**

Mathematically proficient students can apply the mathematics they know to solve problems arising in everyday life, society, and the workplace. In early grades, this might be as simple as writing an addition equation to describe a situation. In middle grades, a student might apply proportional reasoning to plan a school event or analyze a problem in the community. By high school, a student might use geometry to solve a design problem or use a function to describe how one quantity of interest depends on another. Mathematically proficient students who can apply what they know are comfortable making assumptions and approximations to simplify a complicated situation, realizing that these may need revision later. They are able to identify important quantities in a practical situation and map their relationships using such tools as diagrams, two-way tables, graphs, flowcharts and formulas. They can analyze those relationships mathematically to draw conclusions. They routinely interpret their mathematical results in the context of the situation and reflect on whether the results make sense, possibly improving the model if it has not served its purpose.

#### **5 Use appropriate tools strategically.**

Mathematically proficient students consider the available tools when solving a mathematical problem. These tools might include pencil and paper, concrete models, a ruler, a protractor, a calculator, a spreadsheet, a computer algebra system, a statistical package, or dynamic geometry software. Proficient students are sufficiently familiar with tools appropriate for their grade or course to make sound decisions about when each of these tools might be helpful, recognizing both the insight to be gained and their limitations. For example, mathematically proficient high school students analyze graphs of functions and solutions generated using a graphing calculator. They detect possible errors by strategically using estimation and other mathematical knowledge. When making mathematical models, they know that technology can enable them to visualize the results of varying assumptions, explore consequences, and compare predictions with data. Mathematically proficient students at various grade levels are able to identify relevant external mathematical resources, such as digital content located on a website, and use them to pose or solve problems. They are able to use technological tools to explore and deepen their understanding of concepts.

#### **6 Attend to precision.**

Mathematically proficient students try to communicate precisely to others. They try to use clear definitions in discussion with others and in their own reasoning. They state the meaning of the symbols they choose, including using the equal sign consistently and appropriately. They are careful about specifying units of measure, and labeling axes to clarify the correspondence with quantities in a problem. They calculate accurately and efficiently, express numerical answers with a degree of precision appropriate for the problem context. In the elementary grades, students give carefully formulated explanations to each other. By the time they reach high school they have learned to examine claims and make explicit use of definitions.

**7 Look for and make use of structure.**

Mathematically proficient students look closely to discern a pattern or structure. Young students, for example, might notice that three and seven more is the same amount as seven and three more, or they may sort a collection of shapes according to how many sides the shapes have. Later, students will see  $7 \times 8$  equals the well remembered  $7 \times 5 + 7 \times 3$ , in preparation for learning about the distributive property. In the expression  $x^2 + 9x + 14$ , older students can see the 14 as  $2 \times 7$  and the 9 as  $2 + 7$ . They recognize the significance of an existing line in a geometric figure and can use the strategy of drawing an auxiliary line for solving problems. They also can step back for an overview and shift perspective. They can see complicated things, such as some algebraic expressions, as single objects or as being composed of several objects. For example, they can see  $5 - 3(x - y)^2$  as 5 minus a positive number times a square and use that to realize that its value cannot be more than 5 for any real numbers  $x$  and  $y$ .

**8 Look for and express regularity in repeated reasoning.**

Mathematically proficient students notice if calculations are repeated, and look both for general methods and for shortcuts. Upper elementary students might notice when dividing 25 by 11 that they are repeating the same calculations over and over again, and conclude they have a repeating decimal. By paying attention to the calculation of slope as they repeatedly check whether points are on the line through (1, 2) with slope 3, middle school students might abstract the equation  $(y - 2)/(x - 1) = 3$ . Noticing the regularity in the way terms cancel when expanding  $(x - 1)(x + 1)$ ,  $(x - 1)(x^2 + x + 1)$ , and  $(x - 1)(x^3 + x^2 + x + 1)$  might lead them to the general formula for the sum of a geometric series. As they work to solve a problem, mathematically proficient students maintain oversight of the process, while attending to the details. They continually evaluate the reasonableness of their intermediate results.

**Connecting the Standards for Mathematical Practice to the Standards for Mathematical Content**

The Standards for Mathematical Practice describe ways in which developing student practitioners of the discipline of mathematics increasingly ought to engage with the subject matter as they grow in mathematical maturity and expertise throughout the elementary, middle and high school years. Designers of curricula, assessments, and professional development should all attend to the need to connect the mathematical practices to mathematical content in mathematics instruction.

The Standards for Mathematical Content are a balanced combination of procedure and understanding. Expectations that begin with the word “understand” are often especially good opportunities to connect the practices to the content. Students who lack understanding of a topic may rely on procedures too heavily. Without a flexible base from which to work, they may be less likely to consider analogous problems, represent problems coherently, justify conclusions, apply the mathematics to practical situations, use technology mindfully to work with the mathematics, explain the mathematics accurately to other students, step back for an overview, or deviate from a known procedure to find a shortcut. In short, a lack of understanding effectively prevents a student from engaging in the mathematical practices.

In this respect, those content standards which set an expectation of understanding are potential “points of intersection” between the Standards for Mathematical Content and the Standards for Mathematical Practice. These points of intersection are intended to be weighted toward central and generative concepts in the school mathematics curriculum that most merit the time, resources, innovative energies, and focus necessary to qualitatively improve the curriculum, instruction, assessment, professional development, and student achievement in mathematics.